Chapter 7 · Coordinate Geometry

WORK IT OUT

A maths workbook — concepts, worked examples, graded practice and number puzzles

Do the Learn First pages before the practice. Do the practice before the puzzles. Full solutions are at the back — but they only help you after you have tried.

Part	What it is	How to use it	Time
1	Learn First	Read and annotate. Copy every worked example by hand.	25 min
2	Worked Examples	Cover the solution. Try. Then compare line by line.	20 min
3	Guided Practice	Steps are given; you fill the gaps.	15 min
4	Practice Set A · Fluency	Straight application. Aim for speed and accuracy.	20 min
5	Practice Set B · Reasoning	Multi-step. Show every line.	25 min
6	Practice Set C · Challenge	Stretch problems. One may beat you — that is fine.	25 min
7	The Crossnumber	Answers are numbers. Wrong maths will not fit.	20 min
8	Number Search, Magic Square & Concept Bingo	Solo or with a partner.	20 min
9	Spot the Mistake	Audit a worked solution. Find the planted error.	15 min
10	Quick-Fire Round	Matching, True/False, Two Truths and a Lie.	15 min
11	The Escape Challenge	Teams of 4.	25 min

Part 1 · Learn First

Every point on a flat page has an address — an x-coordinate and a y-coordinate. This chapter is about two questions only: how far apart are two addresses, and what address lies between them in a chosen ratio? Everything else — squares, triangles, parallelograms, collinear points — falls out of answering those two questions honestly.

The Distance Formula

To find the distance between two points, draw the right triangle that Pythagoras gave us and let algebra do the measuring.

For a point on the x-axis, distance is just subtraction: A(4, 0) and B(6, 0) are AB = 6 − 4 = 2 units apart. The same works on the y-axis.

For two points **not** on an axis, drop perpendiculars to the axes to build a right triangle, then apply the Pythagoras theorem to the triangle PTQ formed.

$$PQ=\sqrt{}$$

Distance from the origin. The distance of a point P(x, y) from the origin O(0, 0) is a special case with $(x_1, y_1)=(0, 0)$:

$$OP=\sqrt{x^2+y^2}$$

Distance is always taken as the **positive** square root — a length can never be negative, however the subtraction inside the bracket comes out.

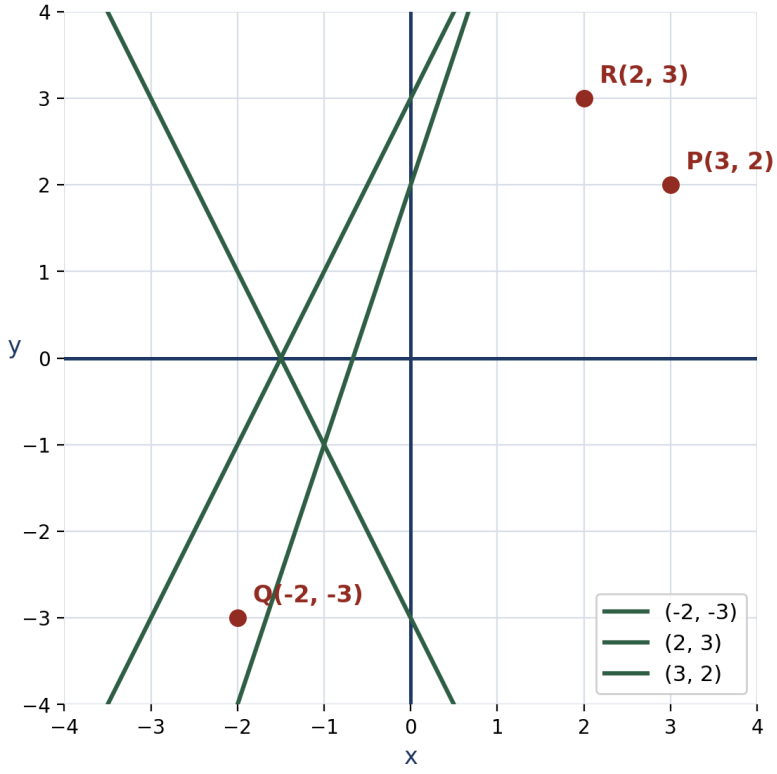

Fig. 7.6 — P(3, 2), Q(−2, −3) and R(2, 3) joined to test for a triangle

Rule to remember

- $PQ^2 = ¿$ is symmetric: it makes no difference which point you call (x_1, y_1) and which (x_2, y_2).
- Three points are **collinear** if the sum of the two smaller distances equals the largest distance.
- Three points form a **triangle** if the sum of any two distances is strictly greater than the third — and it is right-angled if the converse of Pythagoras theorem is satisfied by the three squared distances.

□ Are (3, 2), (−2, −3) and (2, 3) the corners of a triangle?

$PQ = \sqrt{¿¿}$

$QR = \sqrt{¿¿}$

$PR = \sqrt{¿¿}$

Every pair-sum beats the third side, so P, Q, R form a triangle. Also $PQ^2 + PR^2 = 50 + 2 = 52 = QR^2$, so by the converse of Pythagoras theorem $\angle P = 90°$ — a **right** triangle.

The Section Formula

The distance formula tells you *how far apart* two points are. The section formula tells you exactly *which point* lies between them, in whatever ratio you choose.

Section formula. The point P(x, y) which divides the join of A(x_1, y_1) and B(x_2, y_2) internally in the ratio $m_1 : m_2$ has coordinates:

$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

Ratio written as k : 1. If P divides AB in the ratio $k : 1$, this simplifies to a one-parameter form that is often quicker for "find the ratio" problems:

$$\left(\frac{k x_2 + x_1}{k + 1}, \frac{k y_2 + y_1}{k + 1}\right)$$

Mid-point — a special case. The mid-point divides AB in the ratio 1 : 1. Substituting $m_1 = m_2 = 1$ collapses the formula to a simple average:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Trisection points split a segment into three equal parts — the nearer one divides AB in the ratio 1 : 2, the farther one in the ratio 2 : 1.

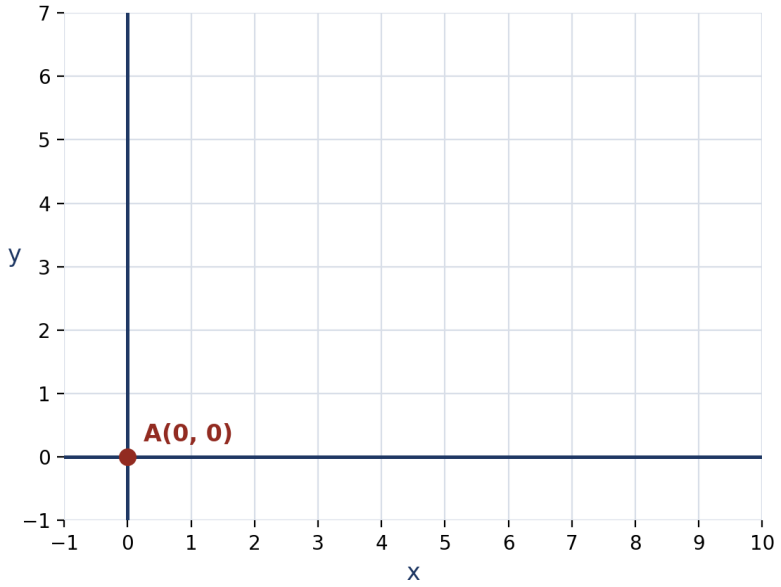

Fig. 7.9 — the relay tower P divides AB in the ratio 1 : 2

Rule to remember

- The diagonals of a **parallelogram bisect each other** — so their mid-points are the same point. This is often the fastest way to find a missing vertex.
- To find a ratio when only the dividing point is known, set the section-formula x-coordinate (or y-coordinate) equal to the known value and solve

for k or for $m_1 : m_2$ — then check with the other coordinate.

- A point that lies **on the x-axis** has y-coordinate 0; a point on the **y-axis** has x-coordinate 0. Setting the section-formula coordinate to 0 finds exactly where a segment crosses that axis.

Find the ratio in which (−4, 6) divides A(−6, 10) and B(3, −8)

Let the ratio be $m_1 : m_2$. By the section formula, $-4 = \frac{3m_1 - 6m_2}{m_1 + m_2}$ and $6 = \frac{-8m_1 + 10m_2}{m_1 + m_2}$.

From the x-equation: $-4m_1 - 4m_2 = 3m_1 - 6m_2 \Rightarrow 7m_1 = 2m_2 \Rightarrow m_1 : m_2 = 2 : 7$.

Checking with the y-equation confirms the same ratio.

Answer: $(-4, 6)$ divides AB in the ratio **2 : 7**.

Part 2 · Worked Examples

Cover the right-hand column with a sheet of paper. Try the problem yourself first. Then uncover it one line at a time and check where your reasoning parted from the model.

Example 1 — Show that (1, 7), (4, 2), (−1, −1) and (−4, 4) are the vertices of a square (Example 2)

Step 1. Let A(1, 7), B(4, 2), C(−1, −1), D(−4, 4). Find all four sides using the distance formula.

$AB=\sqrt{\square}$

$BC=\sqrt{\square}$

$CD=\sqrt{\square}$

$DA=\sqrt{\square}$

Step 2. Find both diagonals.

$AC=\sqrt{\square}$

$BD=\sqrt{\square}$

Answer: All four sides are equal ($\sqrt{34}$) and both diagonals are equal ($\sqrt{68}$), so ABCD is a **square**.

> **□ Why this works**
>
> All sides equal only proves a rhombus; you also need the diagonals equal (or one right angle) to rule out a non-square rhombus. That second check is what separates this proof from an incomplete one.

Example 2 — Find the coordinates of the point dividing (4, −3) and (8, 5) in the ratio 3 : 1 internally (Example 6)

Step 1. Let P(x, y) be the required point, with $A(4,-3)$, $B(8,5)$, $m_1:m_2=3:1$.

Step 2. Apply the section formula for x: $x=\frac{3(8)+1(4)}{3+1}=\frac{28}{4}=7$.

Step 3. Apply it for y: $y=\frac{3(5)+1(-3)}{3+1}=\frac{12}{4}=3$.

Answer: P = **(7, 3)**.

> **□ Why this works**
>
> The pattern to memorise: the coordinate nearer B (the "3" side of the ratio) is weighted by m_1 and multiplies B's coordinate; the one nearer A is weighted by m_2 and multiplies A's coordinate.

Example 3 — Find the coordinates of the points of trisection of A(2, −2) and B(−7, 4) (Example 8)

Step 1. Let P, Q trisect AB, so $AP = PQ = QB$. Then P divides AB in the ratio 1 : 2, and Q divides AB in the ratio 2 : 1.

Step 2. For P (ratio 1:2): $x = \frac{1(-7)+2(2)}{1+2} = \frac{-3}{3} = -1$, $y = \frac{1(4)+2(-2)}{1+2} = \frac{0}{3} = 0$. So $P = (-1, 0)$.

Step 3. For Q (ratio 2:1): $x = \frac{2(-7)+1(2)}{2+1} = \frac{-12}{3} = -4$, $y = \frac{2(4)+1(-2)}{2+1} = \frac{6}{3} = 2$. So $Q = (-4, 2)$.

Answer: The trisection points are **(−1, 0)** and **(−4, 2)**.

> **Why this works**
>
> Q could also have been found faster as the mid-point of P and B — once you have one trisection point, the other is often only a mid-point calculation away.

Example 4 — If A(6, 1), B(8, 2), C(9, 4), D(p, 3) are vertices of a parallelogram (in order), find p (Example 10)

Step 1. The diagonals of a parallelogram bisect each other, so the mid-point of AC equals the mid-point of BD.

Step 2. Mid-point of AC ¿ $\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$.

Step 3. Mid-point of BD ¿ $\left(\frac{8+p}{2}, \frac{2+3}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$.

Step 4. Equate x-coordinates: $\frac{15}{2} = \frac{8+p}{2} \Rightarrow p = 7$.

Answer: $p = 7$.

> **Why this works**
>
> No need to write out both mid-point coordinates in full — the y-coordinates were equal before you even started, so only the x-equation carries information here.

Part 3 · Guided Practice

The method is laid out for you. Fill in the blanks. If you can do these without looking back at Part 2, you are ready for Part 4.

1. Find a point on the y-axis equidistant from A(6, 5) and B(−4, 3) (Example 5 pattern).

A point on the y-axis has the form P(0, y). Set $AP^2 = BP^2$: ¿

Expand both sides: $36 + 25 - 10y + y^2 =$ ¿ $+ 9 - 6y + y^2$

Simplify and solve: $4y =$ ¿ ¿, so $y =$ ¿ _____ . The required point is ¿ _____ ¿.

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2. In what ratio does the point (−1, 6) divide the segment joining (−3, 10) and (6, −8)?

Let the ratio be $k : 1$. By the section formula, the x-coordinate gives $-1 = k$ ¿ ¿

Cross-multiply: $-k - 1 =$ ¿ $k - 3$, so ¿ $_k = 2$, giving $k =$ ¿ _____ .

Ratio ¿ $k : 1 =$ ¿ _____ : 1 ¿ _____ : _____ (in whole numbers).

---

3. Find the distance between the points (0, 0) and (36, 15), and hence the distance between towns A and B of Section 7.2.

Apply the distance formula with $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (36, 15)$: $d = \sqrt{}$ ¿ ¿

Compute the squares: $d = \sqrt{1296 +}$ ¿ ¿ $\sqrt{1521}$ ¿

Take the square root: $d =$ ¿ _____ km.

Practice Set A · Fluency

Straight application of the method. Aim to get every one right, then aim to get them right quickly.

#	Question	Your working and answer
1	Find the distance between the pairs of points: (i) (2, 3), (4, 1) (ii) (−5, 7), (−1, 3) (iii) (a, b), (−a, −b).	
2	Find the distance between the points (0, 0) and (36, 15). Hence find the distance between the towns A and B discussed in Section 7.2.	
3	Determine if the points (1, 5), (2, 3) and (−2, −11) are collinear.	
4	Check whether (5, −2), (6, 4) and (7, −2) are the vertices of an isosceles triangle.	
5	Find the point on the x-axis which is equidistant from (2, −5) and (−2, 9).	
6	Find the values of y for which the distance between P(2, −3) and Q(10, y) is 10 units.	
7	Find the coordinates of the point which divides the join of (−1, 7) and (4, −3) in the ratio 2 : 3.	
8	Find the coordinates of the points of trisection of the line segment joining (4, −1) and (−2, −3).	

Practice Set B · Reasoning

Two or more steps, or a step you have to choose for yourself. Show every line — marks live in the working.

#	Question	Your working and answer
1	In a classroom, 4 friends are seated at A(3, 1), B(6, 4), C(8, 6) and D(5, 3). Champa asks Chameli, "Don't you think ABCD is a square?" Use the distance formula to decide who is right.	
2	If Q(0, 1) is equidistant from P(5, −3) and R(x, 6), find the values of x. Also find the distances QR and PR.	
3	Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (−3, 4).	
4	Find the ratio in which the line segment joining the points (−3, 10) and (6, −8) is divided by (−1, 6).	
5	Find the ratio in which the line segment joining A(1, −5) and B(−4, 5) is divided by the x-axis. Also find the coordinates of the point of division.	
6	If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.	

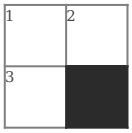
Practice Set C · Challenge

Harder, and deliberately so. If one defeats you, write down exactly where you got stuck. That sentence is worth more than a copied answer.

#	Question	Your working and answer
1	Name the type of quadrilateral formed, if any, by the points (−1, −2), (1, 0), (−1, 2), (−3, 0), and give reasons for your answer.	
2	100 flower pots have been placed at 1 m intervals along AD of a rectangular school ground. Niharika runs 1/4 the distance AD on the 2nd line and posts a green flag; Preet runs 1/5 the distance AD on the 8th line and posts a red flag. Find the distance between the two flags, and where Rashmi should post a blue flag exactly halfway between them.	
3	Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, −3) and B is (1, 4).	
4	If A(−2, −2) and B(2, −4), find the coordinates of P such that $AP = \frac{3}{7} AB$ and P lies on the line segment AB.	
5	Find the area of a rhombus if its vertices, taken in order, are (3, 0), (4, 5), (−1, 4) and (−2, −1). [Hint: area of a rhombus ¿ $\frac{1}{2}$ (product of its diagonals).]	

Part 7 · The Crossnumber

Every answer is a number. One digit per square. Because answers cross, a wrong calculation simply will not fit — this puzzle marks itself.



ACROSS

1. x-coordinate of the relay tower P, dividing AB in the ratio 1 : 2 (Section 7.2) *(2 digits)*

3. y-coordinate of the same relay tower P *(1 digits)*

DOWN

1. Number of km town B is north of town A *(2 digits)*

2. m1, the first term of the ratio 2 : 7 found in Example 7 *(1 digits)*

Bonus questions that did not fit the grid — work them out anyway: Number of km town B is east of town A; x-coordinate of the point dividing (4, −3) and (8, 5) in the ratio 3 : 1 (Example 6); y-coordinate of that same point (Example 6); AB^2 for the square ABCD in Example 2 (side length squared); AC^2 for the square ABCD in Example 2 (diagonal length squared); y-coordinate of the point on the y-axis equidistant from (6, 5) and (−4, 3) (Example 5); Three times the y-coordinate found for the point of intersection with the y-axis in Example 9 (y = 13/3); Size, ignoring sign, of the x-coordinate of the farther trisection point Q(−4, 2) in Example 8

Part 8 · Number Search & Magic Square

Work out each answer below, then find it hidden in the digit grid — across, down, diagonally or backwards.

1. x-coordinate of the relay tower P (Section 7.2)
2. y-coordinate of the relay tower P
3. Distance in km town B is east of town A
4. Distance in km town B is north of town A
5. x-coordinate of the point dividing (4,−3),(8,5) in ratio 3:1 (Example 6)
6. AB^2 for the square in Example 2
7. y-coordinate of the point equidistant from (6,5) and (−4,3) (Example 5)
8. Size of the x-coordinate of trisection point Q in Example 8

0	1	6	1	8	4	5	6	9	9	5	9	0
8	9	3	8	6	1	0	0	8	5	1	9	3
0	5	5	8	2	0	0	3	4	3	8	4	9
0	4	4	3	7	2	3	4	9	0	7	7	1
8	0	4	3	1	7	2	4	5	5	0	1	2
1	9	2	8	3	8	4	3	7	0	9	2	8
4	2	3	1	2	6	9	8	8	0	5	9	4
5	1	4	9	0	3	2	6	7	7	5	1	3
1	2	1	4	0	3	3	7	9	7	5	8	4
9	1	0	8	3	4	0	4	9	8	0	4	9
7	3	5	9	5	3	5	1	5	7	6	3	1
7	9	9	8	2	9	1	4	5	0	7	2	0
5	5	7	9	9	1	7	2	2	5	3	2	2

☐ 12	☐ 36	☐ 15	☐ 34	☐ 5
☐ 7	☐ 9	☐ 4		

Magic Square — every row, column and diagonal adds to the same total.

A 3×3 grid using the consecutive integers 1–9, one per box, every row, column and diagonal summing to the same magic constant.

8		**6**

3	**5**	
	9	

Magic constant: 15. Fill the empty cells.

Concept Bingo

Fill the empty grid below with any 16 terms from the bank — in any order, one per box. Then work down the clue list: each clue describes one term without naming it. Cross off a box only when you are sure which term its clue is describing. Get four crossed in a row, column or diagonal and you have bingo.

Term bank — choose any 16:

Distance formula	Section formula	Mid-point formula	Abscissa
Ordinate	Collinear points	Origin	Coordinate axes
Trisection points	Quadrant	Right triangle	Isosceles triangle
Square	Rhombus	Parallelogram	Perpendicular bisector
Internal division	Pythagoras theorem		

Your bingo card:

Clue list — read each clue and find the term it describes:

① The formula that gives the length of the segment joining two points.
② The formula that gives the coordinates of a point dividing a segment in a given ratio.
③ The special case of the section formula where the ratio is 1 : 1.
④ The x-coordinate of a point.
⑤ The y-coordinate of a point.
⑥ Points that all lie on one straight line.
⑦ The fixed point (0, 0) from which coordinates are measured.
⑧ The pair of perpendicular lines used to locate points on a plane.
⑨ The two points that split a line segment into three equal parts.
⑩ One of the four regions the coordinate axes divide the plane into.

- A triangle with one angle equal to 90°.
- A triangle with (at least) two equal sides.
- A quadrilateral with all sides equal and all diagonals equal.
- A quadrilateral with all sides equal, but diagonals not necessarily equal.
- A quadrilateral whose diagonals bisect each other.
- The line through the mid-point of a segment, at right angles to it — every point on it is equidistant from the two ends.
- Dividing a segment so the dividing point lies between the two given points (as opposed to outside them).
- The theorem $a^2+b^2=c^2$ that the distance formula is built from.

Part 9 · Spot the Mistake

Each solution below reaches a wrong answer, and exactly one step is to blame. Find the first wrong step and rewrite it correctly. Marking other people's work is the fastest way to stop making their mistakes.

□ Spot the Mistake

Finding the distance between (2, 3) and (4, 1)

Step 1. Substitute into the distance formula: $d = \sqrt{¿¿}$.

Step 2. $¿\sqrt{2^2 + ¿¿}$.

Step 3. So the two points are the same point.

_____ The first wrong step is Step _____ . What should it say?

_____

□ Spot the Mistake

Deciding whether three points are collinear

Step 1. For P(1, 5), Q(2, 3), R(−2, −11): $PQ = \sqrt{1+4} = \sqrt{5}$, $QR = \sqrt{16+196} = \sqrt{212}$, $PR = \sqrt{9+256} = \sqrt{265}$.

Step 2. Since $\sqrt{5} + \sqrt{212} \neq \sqrt{265}$ exactly by inspection, the points are not collinear.

Step 3. So P, Q, R form a triangle.

_____ The first wrong step is Step _____ . What should it say?

_____

□ Spot the Mistake

Using the section formula for a ratio written as m1 : m2

Step 1. P divides A(−6,10) and B(3,−8) in the ratio $m_1 : m_2$, so $x = \dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$.

Step 2. Substituting, the x-coordinate formula becomes $x = \dfrac{m_1(-6) + m_2(3)}{m_1 + m_2}$.

Step 3. So a point nearer A gets the larger share of m_1.

_____ The first wrong step is Step _____ . What should it say?

_____

Part 10 · Quick-Fire Round

Match each term to its meaning

Draw a line from each item on the left to its match on the right.

#	Left	Right (shuffled)
1	Distance formula	The x-coordinate of a point
2	Section formula	The y-coordinate of a point
3	Mid-point formula	Gives the length of the segment joining two points
4	Abscissa	Gives the coordinates of a point dividing a segment in a given ratio
5	Ordinate	Points that lie on the same straight line
6	Collinear points	The section formula when the ratio is 1 : 1
7	Trisection points	The two points that divide a segment into three equal parts

True or False Sprint

Read each statement to yourself (or take turns with a partner). Decide True or False before you check the solutions.

#	Statement	Your call
1	The distance formula gives $PQ = \sqrt{}$.	☐ True ☐ False
2	Distance can sometimes come out negative if the coordinates are negative.	☐ True ☐ False
3	The distance of P(x, y) from the origin is $\sqrt{x^2 + y^2}$.	☐ True ☐ False
4	Three points are collinear if the sum of any two of the three distances between them exceeds the third.	☐ True ☐ False
5	The mid-point formula is the section formula with ratio 1 : 1.	☐ True ☐ False
6	The section formula for ratio $m_1 : m_2$ weights $B(x_2, y_2)$ by m_1 and $A(x_1, y_1)$ by m_2.	☐ True ☐ False
7	The diagonals of every parallelogram bisect each other.	☐ True ☐ False
8	A point on the x-axis always has x-coordinate 0.	☐ True ☐ False

#	Statement	Your call
9	Trisection points divide a segment into three equal parts, at ratios 1 : 2 and 2 : 1 from the same starting point.	☐ True ☐ False
10	If all four sides of a quadrilateral are equal, it must be a square.	☐ True ☐ False

Two Truths and a Lie

Two statements are true, one is false. Circle the lie — then correct it.

Set 1

(a) The distance formula uses the Pythagoras theorem.

(b) A negative value under the square root in the distance formula means the points do not exist.

(c) The distance between two points on the x-axis is just the difference of their x-coordinates.

The lie is _____ . Corrected: _____

Set 2

(a) The mid-point formula is a special case of the section formula.

(b) The section formula only works when the ratio is a whole number.

(c) Trisection points can be found using the section formula twice.

The lie is _____ . Corrected: _____

Set 3

(a) All sides equal in a quadrilateral proves it is a rhombus.

(b) All sides equal AND both diagonals equal proves a quadrilateral is a square.

(c) All sides equal AND one angle 90° proves a quadrilateral is a square.

The lie is _____ . Corrected: _____

Part 11 · The Escape Challenge

Five gates, each a calculation. Each gives you one digit. Collect all five to unlock the exit code.

🔒 GATE 1 — The Distance Gate

Find the distance between (0, 0) and (36, 15), in km.

Digit: ☐

🔒 GATE 2 — The Section Gate

Find the x-coordinate of the point dividing (4, −3) and (8, 5) in the ratio 3 : 1.

Digit: ☐

🔒 GATE 3 — The Y-Axis Gate

Find the y-coordinate of the point on the y-axis equidistant from (6, 5) and (−4, 3).

Digit: ☐

🔒 GATE 4 — The Parallelogram Gate

A(6,1), B(8,2), C(9,4), D(p,3) form a parallelogram. Find p.

Digit: ☐

🔒 GATE 5 — The Ratio Gate

The point (−4, 6) divides A(−6,10) and B(3,−8) in the ratio 2 : m2. Find m2.

Digit: ☐

EXIT CODE: ☐ ☐ ☐ ☐ ☐

Formula Sheet

Cut this page out and keep it. Everything in this chapter, on one page.

Distance formula

$$PQ=\sqrt{\square\square}$$

Distance from the origin

$$OP=\sqrt{x^2+y^2}$$

Section formula (ratio m1 : m2)

$$\left(\frac{m_1 x_2+m_2 x_1}{m_1+m_2},\frac{m_1 y_2+m_2 y_1}{m_1+m_2}\right)$$

Section formula (ratio k : 1)

$$\left(\frac{k x_2+x_1}{k+1},\frac{k y_2+y_1}{k+1}\right)$$

Mid-point formula

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Full Solutions

Not just answers — working. Compare your method with this one; if they differ but both reach the right answer, make sure you understand why yours also works.

Part 3 · Guided Practice

1. 4y = 36, y = 9, point (0, 9) · **2.** k = 2/7, ratio = 2 : 7 · **3.** d = √1521 = 39 km

Practice Set A

1. (i) √¿¿. (ii) √¿¿. (iii) √¿¿.

2. $\sqrt{36^2+15^2}=\sqrt{1296+225}=\sqrt{1521}=39$. The towns A and B are 39 km apart.

3. $PQ=\sqrt{1+4}=\sqrt{5}$, $QR=\sqrt{16+196}=\sqrt{212}$, $PR=\sqrt{9+256}=\sqrt{265}$; since $\sqrt{5}+\sqrt{212}\neq\sqrt{265}$ (comparing squares shows no exact sum), the points are **not** collinear.

4. $AB=\sqrt{1+36}=\sqrt{37}$, $BC=\sqrt{1+36}=\sqrt{37}$, $CA=\sqrt{4+0}=2$; $AB=BC$, so it **is** an isosceles triangle.

5. Let P(x, 0). ¿. Point: **(−7, 0)**.

6. ¿ or $y=-9$.

7. $x=\frac{2(4)+3(-1)}{5}=1$, $y=\frac{2(-3)+3(7)}{5}=3$. Point: **(1, 3)**.

8. Ratio 1:2 point: $\left(\frac{1(-2)+2(4)}{3},\frac{1(-3)+2(-1)}{3}\right)=$¿; ratio 2:1 point: ¿.

Practice Set B

1. $AB=\sqrt{9+9}=3\sqrt{2}$, $BC=\sqrt{4+4}=2\sqrt{2}$, $CD=\sqrt{9+9}=3\sqrt{2}$, $DA=\sqrt{4+4}=2\sqrt{2}$; opposite sides equal but not all four equal, and the diagonals $AC=5\sqrt{2}$, $BD=3\sqrt{2}$ are unequal — so **Chameli is correct**, ABCD is not a square (it is a rectangle-like shape, actually just a parallelogram here).

2. $QP^2=QR^2\Rightarrow 25+16=x^2+25\Rightarrow x=\pm 4$. For $x=4$: $QR=\sqrt{41}$, $PR=\sqrt{2}$; for $x=-4$: $QR=\sqrt{41}$, $PR=\sqrt{82}$.

3. ¿ (equivalently $x-y=-2$ after simplifying correctly gives $3x+y=10$; the required relation is $3x+y=10$).

4. Using the section-formula x-equation with ratio $k:1$:

$-1=\frac{6k-3}{k+1}\Rightarrow -k-1=6k-3\Rightarrow 7k=2\Rightarrow k=\frac{27}{}$❑, so ratio **2 : 7**.

5. Let the x-axis divide AB in ratio $k:1$: $\frac{5k-5}{k+1}=0\Rightarrow k=1$, ratio **1 : 1**; point

¿$\left(\frac{1(-4)+1(1)}{2},0\right)=\left(\frac{-32}{},0\right)$.

6. Diagonals bisect each other: mid-point of (1,2)-(x,6) = mid-point of (4,y)-(3,5).

$\frac{1+x}{2}=\frac{4+3}{2}\Rightarrow x=6;\ \frac{2+6}{2}=\frac{y+5}{2}\Rightarrow y=3.$

Practice Set C

1. All four sides equal $2\sqrt{2}$ and both diagonals equal 4 — equal sides with equal diagonals means the figure is a **square**.

2. Green flag at $(2,25)$, red flag at $(8,20)$ (using $AD=100$m along the 1-unit-per-m grid); distance between flags ¿$\sqrt{36+25}=\sqrt{61}$m; Rashmi's blue flag (mid-point) is at $(5,22.5)$.

3. Centre (2,−3) is the mid-point of AB, so $A=(2(2)-1,2(-3)-4)=(3,-10)$.

4. $AP=\frac{37}{A}B$ means P divides AB in ratio 3:4:

$P=(\frac{3(2)+4(-2)}{7},\frac{3(-4)+4(-2)}{7})=(\frac{-27}{,}-\frac{20}{7}).$

5. Diagonals: $d_1=\sqrt{¿¿}$, $d_2=\sqrt{¿¿}$; area ¿$\frac{12}{(¿4\sqrt{2})(6\sqrt{2})=24¿}$ square units.

Part 7 · Crossnumber — Solved Grid

Across: 1 = 12 · 3 = 5

Down: 1 = 15 · 2 = 2

Part 8 · Number Search — Solved Grid

0	1	6	1	8	4	5	6	9	9	5	9	0
8	9	3	8	6	1	0	0	8	5	1	9	3
0	5	5	8	2	0	0	3	4	3	8	4	9
0	4	4	3	7	2	3	4	9	0	7	7	1
8	0	4	3	1	7	2	**4**	5	5	0	1	2
1	9	2	8	3	8	4	**3**	7	0	9	2	8
4	2	3	1	2	6	9	8	8	0	5	**9**	4
5	1	**4**	9	0	3	2	6	7	**7**	5	1	3
1	**2**	1	4	0	3	3	7	9	7	5	8	4
9	**1**	0	8	3	4	0	4	9	8	0	4	9

7	3	5	9	5	3	5	1	5	7	**6**	**3**	1
7	9	9	8	2	9	1	4	5	0	7	2	0
5	**5**	7	9	9	1	7	2	2	5	3	2	2

Part 8 · Magic Square — Solved (constant 15)

8	**1**	**6**
3	**5**	**7**
4	**9**	**2**

Concept Bingo — Clue Key

#	Clue	Term
1	The formula that gives the length of the segment joining two points.	Distance formula
2	The formula that gives the coordinates of a point dividing a segment in a given ratio.	Section formula
3	The special case of the section formula where the ratio is 1 : 1.	Mid-point formula
4	The x-coordinate of a point.	Abscissa
5	The y-coordinate of a point.	Ordinate
6	Points that all lie on one straight line.	Collinear points
7	The fixed point (0, 0) from which coordinates are measured.	Origin
8	The pair of perpendicular lines used to locate points on a plane.	Coordinate axes
9	The two points that split a line segment into three equal parts.	Trisection points
10	One of the four regions the coordinate axes divide the plane into.	Quadrant
11	A triangle with one angle equal to 90°.	Right triangle
12	A triangle with (at least) two equal sides.	Isosceles triangle

#	Clue	Term
13	A quadrilateral with all sides equal and all diagonals equal.	Square
14	A quadrilateral with all sides equal, but diagonals not necessarily equal.	Rhombus
15	A quadrilateral whose diagonals bisect each other.	Parallelogram
16	The line through the mid-point of a segment, at right angles to it — every point on it is equidistant from the two ends.	Perpendicular bisector
17	Dividing a segment so the dividing point lies between the two given points (as opposed to outside them).	Internal division
18	The theorem $a^2+b^2=c^2$ that the distance formula is built from.	Pythagoras theorem

Part 9 · Spot the Mistake

Problem	First wrong step	It should say
Finding the distance between (2, 3) and (4, 1)	Step 1	Squaring a negative number gives a **positive** result: ¿, not -4. The correct working is $\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$, not 0.
Deciding whether three points are collinear	Step 1	Checking collinearity by eye on irrational square roots is unreliable — square the inequality instead, or compute decimal values ($\sqrt{5}\approx2.24$, $\sqrt{212}\approx14.56$, $\sqrt{265}\approx16.28$: $2.24+14.56=16.80\neq16.28$, so in fact they are **not** collinear — but the reasoning, not the eyeballing, must show it).
Using the section formula for a ratio written as m1 : m2	Step 1	The formula weights each point by the **other** subscript's ratio term: $A(x_1, y_1)$ is multiplied by m_2 and $B(x_2, y_2)$ by m_1. The correct substitution is $x=\dfrac{m_1(3)+m_2(-6)}{m_1+m_2}$ — the labels were swapped.

Part 10 · Matching

#	Left	Correct match
1	Distance formula	Gives the length of the segment joining two points
2	Section formula	Gives the coordinates of a point dividing a segment in a given ratio
3	Mid-point formula	The section formula when the ratio is 1 : 1
4	Abscissa	The x-coordinate of a point
5	Ordinate	The y-coordinate of a point
6	Collinear points	Points that lie on the same straight line
7	Trisection points	The two points that divide a segment into three equal parts

Part 10 · True or False

1-T · 2-F · 3-T · 4-F · 5-T · 6-T · 7-T · 8-F · 9-T · 10-F

Part 10 · Two Truths and a Lie

Set	The lie	Correction
1	b	The expression under the root, ¿, is a sum of two squares — it can never be negative, so this situation never actually arises.
2	b	The ratio $m_1 : m_2$ in the section formula can be any positive real numbers — not just whole numbers — as long as $m_1 + m_2 \neq 0$.
3	a	Equal sides alone give a **rhombus**, not automatically a square — you additionally need equal diagonals, or one right angle, to rule out a "squashed" rhombus.

Part 11 · The Escape Challenge

EXIT CODE: 39 · 7 · 9 · 7 · 7

(Answer: 39) (Answer: 7) (Answer: 9) (Answer: 7) (Answer: 7)

Two questions — how far, and what point lies between — turn out to answer almost every "what shape is this?" question coordinate geometry can ask. Distance and section: that is the whole chapter. □□